SUFFICIENT CONDITIONS OF ENCOUNTER IN A DIFFERENTIAL GAME

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N. N. KRASOVSKII
(Sverdlovsk)
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The differential game of guiding a conflict-controlled motion onto a given set is considered. The sufficient conditions for winning the game are established. These are obtained by extending the conditions previously established for systems with additively separated controls to the case of mixed controls. The study is a continuation of [1-14].

1. Statement of the problem. Let us consider the system described by the equation

$$dx/dt = f(t, x, u, v) \tag{1.1}$$

where x is the n-dimensional phase vector of the system; u, v are the r-dimensional control vectors at the discretion of the first and second players, respectively, and restricted by the conditions

$$u \in P, \qquad v \in Q \tag{1.2}$$

where P and Q are bounded closed sets; the vector function $f(t, x, u, v)$ is continuous in all its arguments and satisfies the Lipschitz conditions in the variable x. The initial position $\{t_0, x_0\}$ and the closed set M in the space $\{x\}$ (i.e. the target) are given. The strategies U and V of the first and second player are defined by the systems of sets $\{\mu(du)\}_{\{t, x\}}$ and $\{\nu(dv)\}_{\{t, x\}}$. These sets consist of the regular normed measures $\mu(du)$ and $\nu(dv)$ on P and Q, respectively. Each possible position $\{t, x\}$ is associated with a set $\{\mu(du)\}_{\{t, x\}}$ which defines U and with a set $\{\nu(dv)\}_{\{t, x\}}$ which defines V. The motion $x[t] = x[t, t_0, x_0; U, V]$ of system (1.1) generated in the interval $[t_0, \vartheta]$ by the strategies U, V from the initial position $\{t_0, x_0\}$ is defined as any function $x[t]$ $(t_0 \leqslant t \leqslant \vartheta)$ which is a uniform limit as $\Delta \to 0$ for the subsequences of continuous Euler broken lines $x_\Delta[t]$ satisfying the equation

$$x_\Delta^{\cdot} = \int_P \int_Q f(t, x_\Delta[t], u, v)\, \mu(du)_{\{\tau_i, x_\Delta[\tau_i]\}} \nu(dv)_{\{\tau_i, x_\Delta[\tau_i]\}} \tag{1.3}$$
$$(\tau_i \leqslant t < \tau_{i+1}, \quad \tau_{i-1} - \tau_i \leqslant \Delta, \quad \tau_0 = t_0, \quad x_\Delta[t_0] = x_0)$$

where $\mu(du)_{\{\tau, x\}}$, $\nu(dv)_{\{\tau, x\}}$ are some (arbitrary) elements from the sets $\{\mu(du)\}_{\{\tau, x\}}$, $\{\nu(dv)\}_{\{\tau, x\}}$.

We say that the strategy U° guarantees the encounter of object (1.1) with the target M by the instant ϑ if every motion $x[t] = x[t, t_0, x_0; U^\circ, V]$ intersects M at least once for $t_0 \leqslant t \leqslant \vartheta$ for every strategy V. The problem consists in determining the conditions under which the strategy U° exists, and in constructing this strategy.

The results of [14] imply that the following alternatives exist for every position $\{t_0, x_0\}$: either there exists a strategy U° which guarantees encounter by a given instant ϑ, or there exists a strategy V° which guarantees the deviation of every motion $x[t] = x[t, t_0, x_0; U, V^\circ]$ from M for all $t_0 \leqslant t \leqslant \vartheta$.

The first of these alternatives is realized if and only if the point x_0 lies in some positional absorption set $W(t_0, \vartheta)$.

Paper [14] contains two ways of describing this set: (1°) is a description based on the notion of positional absorption of the target M; (2°) is a description based on a reversed construction which pushes off from M at the instant ϑ and develops in the direction of decreasing time τ until an instant $t_0 < \vartheta$. Both descriptions are generally ineffective.

We shall formulate sufficient conditions whose fulfillment means that the sets $W(t_0, \vartheta)$ can be replaced by the program absorption sets $W_n(t_0, \vartheta)$ whose initial description is coarser. The constructions previously described for systems with separated controls u and v (e.g. see [11, 12]) can then be extended to Eq. (1.1).

2. Preset absorption sets. We call the strategies $U = U_n$ and $V = V_n$ "preset controls" if the sets $\{\mu\}_{(t,x)}$ and $\{\nu\}_{(t,x)}$ which define them depend only on t, i.e. if they are the sets $\{\mu\ (du)\}_t$ and $\{\nu\ (dv)\}_t$. We say that process (1.1) "preset-absorbs" the target M at an instant $\vartheta \geqslant t_*$ from the position $\{t_*, x_*\}$ if, whatever the preset control V_n, there exists a preset control U_n such that at least one motion $x[t] = x[t, t_0, x_0; U_n, V_n]$ satisfies the condition

$$x[\vartheta] \in M \tag{2.1}$$

(We can confine ourselves to those preset controls V_n which are defined by measures $\nu\ (dv)_t$ piecewise-constant with respect to time).

We define the preset absorption set $W_n\ (t_*, \vartheta)$ (at an instant $\vartheta \geqslant t_*$) as the set of all those points $x = x_*$ for which process (1.1) preset-absorbs the target M at the instant ϑ from the position $\{t_*, x_*\}$.

It is possible to verify that the sets $W_n\ (t, \vartheta)$ are closed for all $t_0 \leqslant t \leqslant \vartheta$. Moreover, $W_n\ (\vartheta, \vartheta) = M$. It is important for our purpose that the sets $W_n\ (t, \vartheta)$ have the following property of strong stability (see [12–14]): for every value $t_* \in [t_0, \vartheta]$, every point $x_* \in W_n\ (t_*, \vartheta)$, every number $\Delta \in (0, \vartheta - t_*]$ and every preset control V_n there exists a preset control U_n such that at least one motion $x[t] = x[t, t_*, x_*; U_n, V_n]$ satisfies the condition

$$x[t_* + \Delta] \in W_n\ (t_* + \Delta, \vartheta) \tag{2.2}$$

In fact, according to [14] fulfillment of the above condition means that the extremal strategy $U^{(e)}$ defined by the sets $\{\mu^{(e)}(du)\}_{(t,x)}$ determined from the maximum condition

$$\min_\nu \Big[\iint\limits_{PQ} s'f(t, x, u, v)\, \mu^{(e)}(du)\, \nu(dv)\Big] = \max_\mu \min_\nu \Big[\iint\limits_{PQ} s'f(t, x, u, v) \mu(du) \nu(dv)\Big] \tag{2.3}$$

guarantees for every motion $x[t] = x[t, t_0, x_0; U^{(e)}, V]$ for x_0 from $W_n\ (t_0, \vartheta)$ the inclusion of $x[t]$ in $W_n\ (t, \vartheta)$ for all $t_0 \leqslant t \leqslant \vartheta$, and thus ensures the required inclusion (2.1). Here the vector $s = x^* - x$, where x^* is the point from $W_n\ (t, \vartheta)$ which lies closest to the point x; the prime denotes transposition.

Note 2.1. It is sufficient that the condition which defines the strong stability property be fulfilled only for all sufficiently small $\Delta > 0$ $(\Delta \leqslant \vartheta - t_*)$ and for preset controls V_n defined by a unique measure $\nu(dv)$ which remains constant in the half-interval $t_* \leqslant t < t_* + \Delta$.

We must therefore find the sufficient conditions whose fulfillment means that the sets $W_n\ (t, \vartheta)$ $(t_0 \leqslant t \leqslant \vartheta)$ are strongly stable. These conditions can be constructed as follows. We use the symbol $X\ (t_*, x_*, t^*, V_n)$ $(t^* = t_* + \Delta)$ to denote the set consisting of all the points $x = x[t^*, t_*, x_*; V_n, U_n]$ resulting from all possible choices of the preset controls $U_n\,(t_* < t^*)$.

Condition 1°. The set $X\,(t_*,\ x_*,\ t^*,\ V_n^\circ)$ is convex for all positions $\{t_*, x_*\}$ $(t_* < \vartheta)$, all sufficiently small numbers $\Delta > 0$, and all preset controls V_n° defined by measures $\nu\,(dv)$ which are constant with respect to time, $t_* \leqslant t < t^* = t_* + \Delta$.

Let the position $\{t^*, x^*\}$ be such that the point x^* is not contained in the set $W_n\,(t^*,\ \vartheta)$ $(t^* \leqslant \vartheta)$. Then there exists a preset control $V_n^*\,(t^* \leqslant t \leqslant \vartheta)$ such that

$$\rho\,(x\,[\vartheta,\ t^*,\ x^*;\ U_n,\ V_n^*],\ M) > \varepsilon\,(x^*) > 0 \tag{2.4}$$

for all preset controls U_n. Here the symbol $\rho\,(x,\ M)$ denotes the distance from the point x to the set M. Further, let the point $x_* \in W_n\,(t_*,\ \vartheta)$ and let us choose some preset control V_n° defined on the half-interval $t_* \leqslant t < t^*$ by a measure $\nu\,(dv)$ constant with respect to the time t. When paired with some preset control $U_n\,(t_* \leqslant t < t^*)$ the chosen preset control V_n° generates some motion $x\,[t] = x\,[t,\ t_*,\ x_*;\ U_n,\ V_n^\circ]$. If the point $x^* = x\,[t^*]$ does not occur in $W_n\,(t^*,\ \vartheta)$, then we continue the control V_n° onto the entire half-interval $[t_*,\ \vartheta)$ by means of the control V_n^* satisfying condition (2.4).

We denote the resulting control V_n by the symbol $V_n^{\circ *}$. But the point x_* occurs in the set $W_n\,(t_*,\ \vartheta)$ and therefore the preset control $V_n^{\circ *}$ for all $\varepsilon > 0$ can be associated with a preset control $U_n^*\,(t_* \leqslant t < \vartheta)$ such that at least one motion $x\,[t] = x\,[t,\ t_*,\ x_*;\ U_n^*,\ V_n^{\circ *}]$ satisfies the condition $x\,[\vartheta] \in M_\varepsilon$, where M_ε — ε is a neighborhood of the set M. The intersection of the set of such motions chosen in some way with the hyperplane $t = t^*$ will be denoted by the symbol $Y_\varepsilon\,(t_*,\ x_*;\ t^*,\ x^*;\ V_n^\circ)$. Let Y_ε^* be the closure of the set Y_ε. The intersection of all Y_ε^* for $\varepsilon > 0$ will be denoted by the symbol $Y^*(t_*,\ x_*;\ t^*,\ x^*;\ V_n^\circ)$.

Condition 2°. The nonempty sets $Y^*\,(t_*,\ x_*;\ t^*,\ x^*;\ V_n^\circ)$ are convex and semicontinuous above relative to inclusion with respect to the variation of x^* in the domain outside $W_n\,(t^*, \vartheta)$ for all sufficiently small values of $\Delta = t^* - t_* > 0$.

The following statement is valid.

Lemma 2.1. If conditions 1° and 2° are fulfilled, the sets $W_n\,(t,\ \vartheta)$ $(t_0 \leqslant t \leqslant \vartheta)$ are strongly stable sets.

In fact, if we assume that the lemma is invalid, then there exists a position $\{t_*,\ x_*\}$, a small number $\Delta > 0$, and a preset control $V_n^\circ\,(t_* \leqslant t < t^* = t_* + \Delta)$ such that the closed sets $W_n(t^*,\ \vartheta)$ and $X(t^*,\ x_*,\ t_*;\ V_n^\circ)$ do not intersect. This in turn would mean that it is possible to construct a mapping $x^* \to Y^*(t_*, x_*;\ t^*,\ x^*;\ V_n^\circ)$ of points x^* from X onto the closed convex subsets Y^* from X.

It is clear that the point x^* cannot occur in the corresponding set Y^*, since this would mean that the preset control V_n^* and some preset control $U_n^*\,(t^* \leqslant t < \vartheta)$ generate from this point a motion $x[t]$ which comes arbitrarily close to the set M at the instant ϑ. But this cannot happen by virtue of our choice of V_n^* from condition (2.4). At the same time, by virtue of the fixed-point theorem of [15], the mapping $x^* \to Y^*$ which we construct necessarily determines one point x° which satisfies the condition $x^\circ \in Y^*$ $(t_*,\ x_*;\ t^*,\ x^\circ;\ V_n^\circ)$. The resulting contradiction proves the lemma.

The following statement follows from the results of [14] and Lemma 2.1 of the present paper.

Theorem 2.1. If Conditions 1° and 2° are fulfilled and if the point x_0 lies in $W_n\,(t_0,\ \vartheta)$, then the extremal strategy $U^{(e)}$ defined by condition (2.3) guarantees the encounter of object (1.1) with the target M by the instant ϑ.

Note 2.2. In dealing with the preset controls U_n and V_n we can avoid the explicit use of limiting transitions from Euler broken lines (1.3) to the motions $x[t]$ by restricting the class of preset controls U_n and V_n by means of appropriate regularizing conditions (e.g. see [16]) and by determining the motions $x[t]$ directly as the solutions of the corresponding limiting generalized differential equations. On the other hand, in determining the motion $x[t]$ directly by taking the limits of Euler broken lines (1.3) it is possible to ignore restrictions on the character of the dependence of $\mu(du)_t$, $\nu(dv)_t$ on t.

3. An intrinsically linear object. Effective description of the sets $W_n(t, \vartheta)$ and effective verification of the conditions of their strong stability are difficult in the general case of a nonlinear equation (1.1). The problem becomes significantly simpler if the right side of Eq. (1.1) does not depend explicitly on the phase vector, i.e. if the equation of motion is of the form

$$dx/dt = f(t, u, v) \tag{3.1}$$

The following case of an intrinsically linear object (1.1) where the equation of motion is linear in z is also reducible to the above case:

$$dz/dt = A(t)z + f(t, u, v) \tag{3.2}$$

In fact, to reduce Eq. (3.2) to (3.1) we need merely make the nonsingular linear substitution of variables $z = Z(t, \vartheta)x$, where $Z(t, \vartheta)$ is the fundamental matrix of solutions of the homogeneous equation $z^{\cdot} = A(t)z$.

Thus, let us consider our problem in the case where the equation of motion is of the form (3.1), where the function $f(t, u, v)$ is continuous with respect to all its arguments. Let us make yet another simplifying assumption; specifically, let us assume that the set M is convex and bounded. The latter condition is not too important, since for a given initial position $\{t_0, x_0\}$ motions (3.1), (3.2) can attain only a bounded portion of M by a finite instant ϑ; this part of M can be taken as the new traget if the initially prescribed set M is unbounded.

The preset absorption set $W_n(t, \vartheta)$ in the case under consideration can be described as follows. Along with Eq. (3.1) we consider the ancillary equation

$$dx/d\tau = f(\tau, u, v) - p\delta(\tau - \vartheta) \tag{3.3}$$

where the symbol $\delta(\tau)$ is a delta function and p is an n-dimensional vector restricted by the condition $p \in M$. Let us choose some preset control V_n defined by the measure $\nu(dv)_\tau$ $(t \leqslant \tau < \vartheta)$. (We can limit ourselves to those preset controls which are defined by measures piecewise-continuous with respect to time τ (see p. 750 and compare Note 2.1)). The definition of the set $W_n(t, \vartheta)$ implies that the point $x = x_*$ belongs to this set if and only if for every V_n there exists a preset control U_n defined by some measure $\mu(du)_\tau$ $(t \leqslant \tau < \vartheta)$ and a vector p such that the motion

$$x(\tau) = x(\tau, t, x_*; U_n, V_n, p)$$

which they generate satisfies the condition $x(\vartheta) = 0$. The motion $x(\tau)$ of system (3.3) is again defined as the uniform limit of Euler broken lines of the form (1.3) which are continuous for $\tau < \vartheta$ but are now constructed for Eq. (3.3) with allowance for the terminal jump $x(\vartheta) - x(\vartheta - 0) = -p$.

The set of all points $q = x(\vartheta)$ generated by the motions

$$x(\tau) = x(\tau, t, x_*, U_n, V_n, p)$$

for a fixed control V_n and for all possible permissible controls U_n and $p \in M$ is called the "attainability domain" with respect to u for the motion $x(\tau)$ (3.3) from the position $\{t, x_*\}$ by the instant ϑ. We shall denote such an attainability domain by the symbol $G(t, x_*, \vartheta; V_n)$. It is a bounded, convex, and closed set.

By virtue of the foregoing, the point x_* lies in the set $W_n(t, \vartheta)$ if and only if the domain $G(t, x_*, \vartheta; V_n)$ contains the point $q = 0$ for every chosen preset control V_n. To describe $W_n(t, \vartheta)$ we must repeat our argument for Eq. (3.1) linear in u and v (e.g. see [12, 17]); we need merely replace the ordinary controls $u(\tau)$ and $v(\tau)$ by the measures $\mu(du)_\tau$ and $\nu(dv)_\tau$ which represent them in this case.

The bounded, convex, and closed set $G(t, x_*, \vartheta; V_n)$ is the intersection [18] of its support half-spaces

$$\varkappa(t, x_*, \vartheta, l; V_n) - l'q \geqslant 0 \tag{3.4}$$

where $\varkappa(t, x_*, \vartheta, l; V_n)$ is the support function of the set G, and l is an arbitrary n-dimensional vector. We have

$$\varkappa(t, x_*, \vartheta, l; V_n) = \max l'q = \qquad q \in G$$

$$= \sup\nolimits_{\mu, p} \left[l' \left(x_* + \int_t^\vartheta \int_P \int_Q f(\tau, u, v)\, \mu(du)_\tau\, \nu(dv)_\tau - p \right) \right] \tag{3.5}$$

where the upper face is taken over all the measures $\mu(du)_\tau$ piecewise-constant with respect to the time τ and where $p \in M$. Thus, the point x_* lies in $W_n(t, \vartheta)$ if and only if the point $q = 0$ satisfies inequality (3.4) for all vectors l and for all measures $\nu(dv)_\tau$ piecewise-continuous with respect to time which define the preset control V_n. We therefore infer from (3.4) and (3.5) that the set $W_n(t, \vartheta)$ is described by the inequality

$$\inf\nolimits_\nu \sup\nolimits_\mu \left[\int_t^\vartheta \int_P \int_Q l'f(\tau, u, v)\, \mu(du)_\tau\, \nu(dv)_\tau \right] + \max\nolimits_p (-l'p) + l'x_* \geqslant 0 \tag{3.6}$$

which must hold for every point x_* from $W_n(t, \vartheta)$ (and only for such points) for every vector l.

We infer from (3.6) that $W_n(t, \vartheta)$ is a bounded, convex, and closed set. Its ε-neighborhood is in turn described by the inequality

$$\|l\|\varepsilon + \inf\nolimits_\nu \sup\nolimits_\mu \left[\int_t^\vartheta \int_P \int_Q l'f(\tau, u, v)\, \mu(du)_\tau\, \nu(dv)_\tau \right] + \max\nolimits_p (-l'p) + l'x_* \geqslant 0$$

$$(\|l\| = (l_1^2, \ldots, l_n^2)^{1/2})$$

This implies that if the point x_* does not lie in the set $W_n(t, \vartheta)$ then its distance $\varepsilon = \rho(x_*, W_n)$ from $W_n(t, \vartheta)$ is defined by the inequality

$$\min\nolimits_{\|l\|=1} (\inf\nolimits_\nu \sup\nolimits_\mu \left[\int_t^\vartheta \int_P \int_Q l'f(\tau, u, v)\, \mu(du)_\tau\, \nu(dv)_\tau \right] +$$

$$+ \max\nolimits_p (-l'p) + l'x_* + \rho(x_*, W_n(t, \vartheta))) = 0 \tag{3.7}$$

The following statement is valid.

Theorem 3.1. If the minimum in the left side of (3.7) under the condition $\rho(x_*, W_n(t, \vartheta)) > 0$ is attained on the unit vector l° and if $x_0 \in W_n(t_0, \vartheta)$, then the extremal strategy $U^{(e)}$ (2.3), where $s = \| x^* - x \| l^\circ$, guarantees the encounter of object (1.1) with the target M.

To prove the theorem we must compute the change in the quantity $\varepsilon_\Delta[t] = \rho(x_\Delta[t], W_n(t, \vartheta))$ in a single step $\Delta = \tau_{i+1} - \tau_i$ along the Euler broken line from that sequence of the latter which in the limiting case defines the motion $x[t] = x[t, t_0, x_0; U^{(e)}, V]$. in which we are interested. In view of the continuous variation of the vector l° with changes in position and the fact that $\mu(du)_{\{\tau_i, x[\tau_i]\}}$ satisfies maximum condition (2.3) for $t = \tau_i$, $x = x_\Delta[\tau_i]$, we obtain the following estimate (see the analogous cases in [12, 17]) from (3.7) for $\varepsilon_\Delta[\tau_i] > 0$:

$$\varepsilon_\Delta[\tau_i + \Delta] - \varepsilon_\Delta[\tau_i] \leqslant o(\Delta) \tag{3.8}$$

where the symbol $\sigma(\Delta)$ denotes an infinitesimal of higher order than Δ. Estimate (3.8) implies that the motion $x[t]$, which is the limit of the curves $x_{\Delta_j}[t]$ as $\Delta_j \to 0$, cannot leave the set $W_n(t, \vartheta)$ as t varies from $t = t_0$ to $t = \vartheta$. Hence, the point $x[\vartheta]$ lies in the set $W_n(\vartheta, \vartheta)$ coincident with M, Q. E. D.

Note 3.1. In this case the vector s in condition (2.3) is unique, since the sets $W_n(t, \vartheta)$ are convex. (As already noted, if the vector l° is unique, it is colinear with the vector s). But this unique vector s varies continuously with variation of the point x This means that the convex sets $\{\mu^{(e)}(du_{\{t,x\}})$ which define the strategy $U^{(e)}$ are weakly semicontinuous above relative to inclusion with respect to the variables x and t (with respect to t on the right, since strongly stable sets $W_n(t, \vartheta)$ are continuous in t on the right (e. g. see [12, 17]). This enables us to formalize the motion $x[t]$ in the form of absolutely continuous solutions of the appropriate differential equations in contingencies

$$x^\cdot[t] \in \int\limits_P\int\limits_Q f(t, x, u, v)\,\mu^{(e)}(du)_{\{t, x[t]\}}\,\nu(dv)_t \tag{3.9}$$

In such cases the general statement corresponding to Theorem 2.1 can be formulated as follows: if the sets $W_n(t, \vartheta)$ $(t_0 \leqslant t \leqslant \vartheta)$ are convex and strongly stable (specifically, if Conditions 1° and 2° are fulfilled) and if the point $x_0 \in W_n(t_0, \vartheta)$, then extremal strategy (2.3) guarantees encounter with M for any motion $x[t] = x[t, t_0, x_0; U^{(e)}, V]$ which is a solution of equation in contingencies (3.9) whatever the strategy V defined by the measure $\nu(dv)_t$ piecewise-continuous (with respect to time t). Theorem 3.1 itself can now be formulated as follows: if $x_0 \in W_n(t_0, \vartheta)$ and if the minimum in the left side of (3.7) is attained on a unique vector l° under the condition $\rho(x_*, W_n(t, \vartheta)) > 0$, then the extremal strategy $U^{(e)}$ (2.3) guarantees encounter with M for every motion $x[t] = x[t, t_0, x_0; U^{(e)}, V]$ which is a solution of equation in contingencies (3.9) whatever the strategy V defined by the measure $\nu(dv)_t$ piecewise-continuous with respect to the time t.

In the general case where the vector s in condition (2.3) is not unique, this formalization runs up against the following obstacle. The sets $\{\mu^{(e)}(du)\}_{\{t,x\}}$ obtained from condition (2.3) may turn out to be nonconvex. If they are complemented up to their convex shells $\{\mu^{(e)}(du)\}^*_{\{t,x\}}$, then, generally speaking, not all of the solutions $x[t]$ of the resulting equations in contingencies (3.9) can be guided onto the set M, but only those solutions (constructive solutions) which are obtainable by taking the limits of Euler broken lines (1.3).

BIBLIOGRAPHY

1. Fleming, W. H., The convergence problem for differential games. J. Math. Anal. and Appl. Vol. 3, №1, 1961.
2. Nardzewski, C. R., A theory of pursuit and evasion. Adv. in Game Theory. Ann. Math. Studies, p. 113, 1964.
3. Pontriagin, L. S., On linear differential games, 2. Dokl. Akad. Nauk SSSR Vol. 175, №4, 1967.
4. Pshenichnyi, B. N., The structure of differential games. Dokl. Akad. Nauk SSSR Vol. 184, №2, 1969.
5. Nikol'skii, M. S., Nonstationary differential games. Vestnik Moskovsk. Gos. Univ. Ser. 1, Matem., Mekh. №3, p. 65, 1969.
6. Varaiya, P. and Lin, J., Existence of saddle points in differential games. SIAM Journal, Control Vol. 7, №1, 1969.
7. Petrov, N. N., On the existence of a pursuit game value. Dokl. Akad. Nauk SSSR Vol. 190, №6, 1970.
8. Smol'iakov, E. R., Differential games in mixed strategies. Dokl. Akad. Nauk SSSR Vol. 191, №1, 1970.
9. Gusiatnikov, P. B., On a certain class of nonlinear differential games. In: Proceedings of the Seminar on the Theory of Optimal Solutions, Academy of Sciences UkrSSR, Scientific Council on Cybernetics, №1, pp. 3-11, Kiev, 1969.
10. Chikrii, A. A., Sufficient conditions of completion of the differential game $r^{\cdot} = ar + f(u, v)$. In: Proceedings of the Seminar on the Theory of Optimal Solutions, Academy of Sciences UkrSSR, Scientific Council on Cybernetics, №3, pp. 17-25, Kiev, 1969.
11. Krasovskii, N. N., On a problem of tracking. PMM Vol. 27, №2, 1963.
12. Krasovskii, N. N. and Subbotin, A. I., A differential homing game. Differentsial'nye Uravneniia Vol. 6, №4, 1970.
13. Krasovskii, N. N., On the theory of differential games. PMM Vol. 34, №2, 1970.
14. Krasovskii, N. N., On a differential convergence game. Dokl. Akad. Nauk SSSR Vol. 193, №2, 1970.
15. Kakutani, S. A., A generalization of Broner's fixed point theorem. Duke Math. J. Vol. 8, 1941.
16. Ioffe, A. D., Generalized solutions of systems with a control. Differentsial'nye Uravneniia Vol. 5, №6, 1969.
17. Krasovskii, N. N., Games problems of dynamics. Izv. Akad. Nauk SSSR, Tekhnicheskaia Kibernetika №1, 1970.
18. Karlin, S., Mathematical Methods in the Theory of Games, Programing and Economics. Moscow, "Mir", 1964.

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